Extended Dynamic Fuzzy Logic System Based on Indirect Adaptive Control for a Class of MIMO Nonlinear Systems

Mohamed Hamdy

Department of Industrial Electronics and Control Engineering

Prince Sultan Industrial Institute, Military Industries Corporation

Al-Kharj, Saudi Arabia

mhamdy72@hotmail.com

***Abstract*-This paper presents an indirect adaptive fuzzy control scheme for a class of unknown multi-input multi-output (MIMO) nonlinear dynamic systems with external disturbances. Within this scheme, the dynamic fuzzy logic system (DFLS) is employed to identify the unknown nonlinear dynamic system. The control law and parameter adaptation laws of the DFLS are derived based on Lyapunov synthesis approach. The control law is robustfied in H^{∞} sense to attenuate external disturbance, model uncertainties, and fuzzy approximation errors. It is shown that under appropriate assumptions, it guarantees the boundness of all signals in the closed-loop system and the asymptotic convergence to zero of tracking errors. Extensive simulation on the tracking control of a two-link rigid robotic manipulator verifies the effectiveness of the proposed algorithms.**

Keywords-MIMO nonlinear system, DFLS, Lyapunov synthesis approach.

I. INTRODUCTION

Identification and control of nonlinear systems have attracted a lot of attention and represents a challenging area in control community during the last two decades. The development of geometric nonlinear control theory and in particular feedback linearization methods, have led to great success in the development of controllers for nonlinear systems [1-2]. A key assumption in these techniques is that the dynamics of the nonlinear systems are exactly known. Some limitations of this theory appear because real systems may have uncertainties. Thus, to deal with uncertain nonlinear systems, many adaptive control approaches have been proposed. Adaptive control approaches are applied to systems with parameter uncertainties. Several results can be found in [3-6] and the references therein.

The introduction of fuzzy systems led to a great success and provides effective approaches to handle nonlinear systems especially complex and ill-defined dynamic systems. Being one of the efficient intelligent techniques, fuzzy systems have been applied to the modeling and control of uncertain nonlinear systems. Based on the universal approximation theorem [7] several stable adaptive fuzzy control schemes have been developed for unknown single-input single-output (SISO) nonlinear systems [7-10], for MIMO nonlinear systems [11-13], and for large-scale interconnected nonlinear systems [14-16] to provide an effective framework for incorporating the expert knowledge systematically and achieve stable performance criterion. The stability analysis in such schemes is performed by using the Lyapunov synthesis approach. However, these adaptive fuzzy control schemes are static in nature. Motivated by the fact that most of physical systems are generally dynamic, this suggests that one may introduce some sort of dynamics to these static fuzzy models in order to cope with the dynamic nature of the physical systems. This would provide a new tool in the control of dynamic systems. The resulting dynamic structure so called the DFLS was first introduced by Lee and Vukovich [17-18]. They have successfully applied this concept to the identification of single-link robotic manipulator [17]. Stable identification and adaptive control based on DFLS was performed in [18]. This work extended to a larger class of SISO nonlinear systems in [19].

However, the previous work on DFLS is limited to only SISO nonlinear systems. Based on the initial results of SISO nonlinear systems [18-19], we intend to develop adaptive dynamic fuzzy control for MIMO nonlinear systems. Furthermore, the proposed scheme is constructed by integrating the feature of H^{∞} tracking performance which can greatly attenuate disturbances, model uncertainties, and fuzzy approximation errors.

The paper is organized as follows. A class of MIMO nonlinear systems and control objectives are described in section 2. Section 3 presents a brief description of static fuzzy systems and DFLS. The DFLS based-adaptive control design is presented in section 4. In section 5, the proposed control algorithm is used to control a two-link robot manipulator. Section 6 concludes this paper.

II. SYSTEM DESCRIPTION

In this paper, we consider a class of MIMO nonlinear system represented by the following set of differential equations:

$$\begin{aligned} y_1^{(n_1)} &= f_1(x) + \sum_{j=1}^{p} g_{1j}(x) u_j + d_1 \\ &\vdots \\ y_p^{(n_p)} &= f_p(x) + \sum_{j=1}^{p} g_{pj}(x) u_j + d_p \end{aligned} \tag{1}$$

where $x = [y_1, \dot{y}_1, \ldots, y_1^{(n_1-1)}, \ldots, y_p, \dot{y}_p, \ldots, y_p^{(n_p-1)}]^T$ is the overall state vector which is assumed to be available for measurements, $u = [u_1, \ldots, u_p]^T$ is the control input vector, $y = [y_1, \ldots, y_p]^T$ is the output vector, $D = [d_1, \ldots, d_p]^T$ denotes the external disturbance, and $f_i(x), g_{ij}(x), i, j = 1, \ldots, p$ are smooth unknown nonlinear functions. Let us denote:

$y^{(n)} = [y_1^{(n_1)} \ldots y_p^{(n_p)}]^T$, $F(x) = [f_1(x) \ldots f_p(x)]^T$,

$$G(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}, \text{ and } D = \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$$

Then, the dynamic system described by (1) can be rewritten in the following compact form:

$$y^{(n)} = F(x) + G(x)u + D \tag{2}$$

Throughout this paper the following assumptions are considered on the system (1)

Assumption 1. The matrix $G(x)$ is bounded away from singularity over compact set $U_c \subset R^n$, specifically

$\|G(x)\|^2 = Trace(G^T(x)G(x)) \geq b_1 \geq 0$, where b_1 represents the smallest singular value of G*(x)*.

Assumption 2. The reference trajectories y_{mi} , $i = 1, \cdots, p$ is known bounded function of time with bounded known derivatives, and it is assumed to be r_i -times differentiable.

Control Objectives: Develop a feedback control law $u(t)$ (based on DFLS) which ensures the boundness of both all variables in the closed-loop systems and the parameters of the DFLS, and guarantees output tracking of a specified desired trajectory $y_{mi} = [y_{m1}, \cdots, y_{mp}]^T$. In addition for a given disturbance attenuation level $\rho > 0$, the following H^∞ tracking performance index is achieved.

$$\begin{aligned} \frac{1}{2}\int_0^T \underline{e}^T Q \underline{e}\ dt &\leq \frac{1}{2}\underline{e}_i^T(0) P_i \underline{e}_i(0) + \frac{1}{2} h_i \tilde{z}^T \tilde{z}(0) \\ &+ \frac{1}{2}\Delta^T(0)\Delta(0) + + \frac{1}{2}\rho^2 \int_0^T \delta^T \delta\, dt \end{aligned} \tag{3}$$

where e is the error vector, $\delta \in L_2[0, T]$ is the combined disturbance and approximation error for $T \in [0, \infty]$, *Q, P* are positive matrices of proper dimensions, Δ is parameter approximation error vector, $\tilde{z}$ is identification error vector , and h is a design parameter.

III. DESCRIPTION OF DFLS

The DFLS is composed of an ordinary fuzzy logic system (also referred to static fuzzy logic system) and a dynamic element. The basic structure of the fuzzy logic system considered in this paper and has been widely used in identification and control of nonlinear systems, is shown in Fig. 1, it is composed of four major components, namely, a fuzzification interface, a fuzzy rule base, a fuzzy inference engine, and a defuzzification interface.

For MIMO fuzzy systems, the fuzzy rule base is made up of the following inference rule:

R^l: If x_1 is F_1^l and x_2 is F_2^l and... x_i is F_i^l Then y_1 is G_1^l and y_2 is G_2^l and… y_j is G_j^l (4)

where F_1^l and G_1^l are fuzzy sets in R , $l = 1, 2, \cdots, N$; $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, p$. Fuzzy inference (4) can be decomposed and expressed as:

R^l: If x_1 is F_1^l and x_2 is F_2^l and... x_n is F_n^l Then y_j is G_j^l $(j = 1, 2, \cdots, p)$

Through center-average defuzzifier, product inference, and singleton fuzzifier [7], the output of the fuzzy logic system can be expressed as:

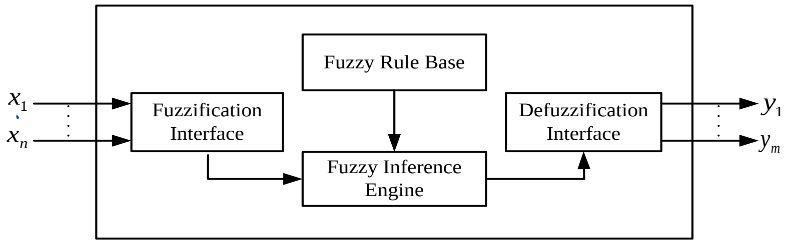

Fig. 1 MIMO Fuzzy Logic System

$$y_j(x) = \frac{\sum_{l=1}^{N} \bar{y}^l \left(\prod_i^n \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{N} \left(\prod_i^n \mu_{F_i^l}(x_i)\right)} \tag{5}$$

where $\bar{y}^l$ is the centre of the fuzzy set G^l at which μ_G^l achieves its maximum value, and we assume that $\mu_G^l(\bar{y}^l) = 1$

Eq. (5) can be written as:

$$y_j(x) = \bar{Y}_j^T \phi(x), \qquad j = 1, 2, \cdots, p \tag{6}$$

where $\bar{Y}_j^T = [\bar{y}_j^1, \cdots, \bar{y}_j^N]^T$ is a vector of adjustable parameters, and $\phi(x) = [\phi_1, \cdots, \phi_N]^T$ is a regression vector with each ϕ_l variable defined as a fuzzy basis function (FBF) [7] as:

$$\phi_l = \frac{\prod_i^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} \left(\prod_i^n \mu_{F_i^l}(x_i)\right)} \tag{7}$$

Define $\bar{Y} = [\bar{Y}_1, \cdots, \bar{Y}_p]_{l \times p}$ as a matrix of adjustable parameters. MIMO fuzzy system can be expressed as:

$$y = \bar{Y}^T \phi(x) \tag{8}$$

The MIMO DFLS shown in Fig. 2 can now be described by the following differential equations:

$$\begin{aligned} \dot{\hat{y}}_1 &= -\alpha_1 \hat{y}_1 + \bar{Y}_1^T \phi(x) \\ &\vdots \\ \dot{\hat{y}}_p &= -\alpha_p \hat{y}_p + \bar{Y}_p^T \phi(x) \end{aligned} \tag{9}$$

Using the definition in (8), equation (9) can be written in the following compact form:

$$\dot{\hat{y}} = -\alpha \hat{y} + \bar{Y}^T \phi(x) \tag{10}$$

where $\hat{y} = [\hat{y}_1, \cdots, \hat{y}_p]$ is the output of MIMO DFLS, $\alpha = diag[\alpha_1, \cdots, \alpha_p]$ is a positive constant matrix.

The DFLS described by (10) was shown to possess universal approximating capabilities to a large class of nonlinear dynamic systems [17].

Let $z \in R^p$ a vector defined as:

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} = \begin{bmatrix} y_1^{(n_1-1)} \\ \vdots \\ y_p^{(n_p-1)} \end{bmatrix} \tag{11}$$

The system (1) can be written as:

$$\begin{aligned} \dot{z}_1 &= f_1(x) + \sum_{j=1}^{p} g_{1j}(x) u_j + d_1 \\ &\vdots \\ \dot{z}_p &= f_p(x) + \sum_{j=1}^{p} g_{pj}(x) u_j + d_p \end{aligned} \tag{12}$$

Or equivalently, equation (12) can be written in the compact form as:

$$\dot{z} = F(x) + G(x)u + D \tag{13}$$

According to the universal approximation theorem [17], the following DFLS can be used to identify the unknown MIMO nonlinear system (12).

$$\dot{\hat{y}} = -\alpha \hat{y} + \bar{Y}^T \phi(x, u) \tag{14}$$

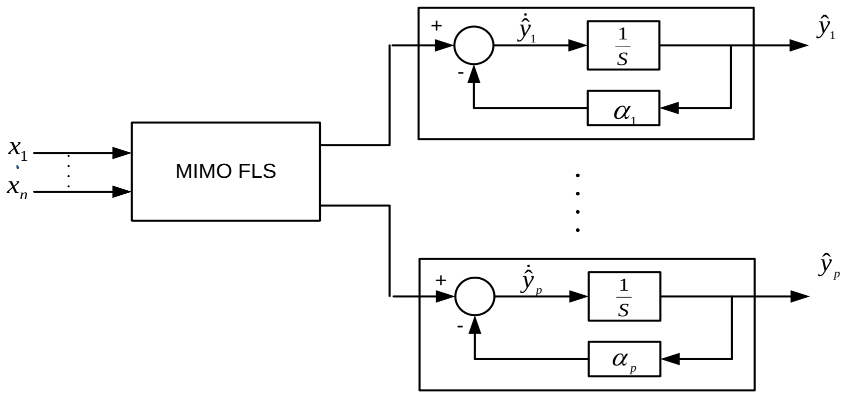

Fig. 2 MIMO Dynamic Fuzzy Logic System

Our objective now is to develop an appropriate control law for input u in (1), and an adaptation law for the parameter matrix $\overline{Y}$ of the DFLS, such that the closed loop system is stable in the sense that the tracking errors $e_j = y_{mj} - y_j$, $j = 1, \cdots, p$, are uniformly bounded and at the same time, the identification errors and identifier parameters are also uniformly bounded.

IV. DFLS Based Adaptive Control

In this section, we develop an adaptive control scheme for system (1) based on DFLS based identification.

Consider the system (1), for the given reference trajectories $y_m = [y_{m1}, \cdots, y_{mp}]^T$. Let us define the tracking errors as:

$$\begin{aligned} e_1 &= y_{m1} - y_1 \\ &\vdots \\ e_p &= y_{mp} - y_p \end{aligned} \tag{15}$$

Denote $e = [e_1, \cdots, e_p]^T$ then $e = y_m - y$

If the system (1) is known i.e. $F(x)$, $G(x)$ are known and $D = 0$, the feedback law:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} = \begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}^{-1} \left(- \begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \right) \tag{16}$$

Yields the linearized systems

$$\begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \tag{17}$$

For reference trajectories to be asymptotically tracked we choose:

$$\begin{aligned} v_1 &= y_{m1}^{(n_1)} + k_{1r_1} e_1^{(n_1 - 1)} + \cdots + k_{11} e_1 \\ &\vdots \\ v_p &= y_{mp}^{(n_p)} + k_{pr_p} e_p^{(n_p - 1)} + \cdots + k_{p1} e_p \end{aligned} \tag{18}$$

Substitute (16) in (1) yields

$$\begin{aligned} e_1^{(n_1)} + k_{1r_1} e_1^{(n_1 - 1)} + \cdots + k_{11} e_1 &= 0 \\ &\vdots \\ e_p^{(n_p)} + k_{pr_p} e_p^{(n_p - 1)} + \cdots + k_{p1} e_p &= 0 \end{aligned} \tag{19}$$

If the coefficients k_{ij} are chosen such that all polynomials in (19) are Hurwitz stable, then we can conclude that $\lim_{t \to \infty} e_i(t) = 0$ which is a main objective of control.

Consider the MIMO DFLS in the form of (9):

$$\begin{aligned} \dot{\hat{z}}_1 &= -\alpha_1 \hat{z}_1 + \overline{Y}_1^T \phi(x, u) \\ &\vdots \\ \dot{\hat{z}}_p &= -\alpha_p \hat{z}_p + \overline{Y}_p^T \phi(x, u) \end{aligned} \tag{20}$$

which can be used to identify the unknown MIMO nonlinear system (12). Define identification errors as $\tilde{z}_i = \hat{z}_i - z_i$, $i = 1, 2, \cdots p$. Our objective is to develop an appropriate control law for input u_i and an adaptive law for the identifier parameters $\overline{Y}_i$ such that closed-loop system is stable.

The expression for $\dot{z}_i$ in equation (12) can be written as

$$\begin{aligned} \dot{z}_1 &= -\alpha_1 z_1 + \overline{Y}_1^T \phi(x,u) - r_1(x,u,\phi,\overline{Y}_1) \\ &\vdots \\ \dot{z}_p &= -\alpha_p z_p + \overline{Y}_p^T \phi(x,u) - r_p(x,u,\phi,\overline{Y}_p) \end{aligned} \tag{21}$$

where $r_i(x,u,\phi,\overline{Y}_i)$ represents the static modeling error of the DFLS identifier and can be expressed as:

$$\begin{aligned} r_i(x,u,\phi,\overline{Y}_i) &= -\alpha_1 z_1 + \overline{Y}_i^T \phi(x,u) \\ &\quad - f_i(x) - \sum_{j=1}^{p} g_{ij}(x) u_j - d_i \end{aligned} \tag{22}$$

By Lemma 1 in [17], there is exist an optimal parameter vectors

$$\overline{Y}_i^* = \min_{\|\overline{Y}_i\|} \{ \overline{Y}_i : \| \overline{Y}_i \| \le M_{\overline{Y}_i} \} \tag{23}$$

which minimize the static modeling error, r_i, such that

$$\sup_{(x,u)\in\Omega} \left| r_i(x,u,\phi,\overline{Y}_i^*) \right| \le M_i^r \tag{24}$$

where $M_{\overline{Y}_i}$ and M_i^r are positive design constants. In the following, we develop an adaptive law for $\overline{Y}_i$. Replace $\overline{Y}_i$ by $\overline{Y}_i^*$ in (21) results in

$$\begin{aligned} \dot{z}_1 &= -\alpha_1 z_1 + \overline{Y}_1^{*T} \phi(x,u) - r_1(x,u,\phi,\overline{Y}_1^*) \\ &\vdots \\ \dot{z}_p &= -\alpha_p z_p + \overline{Y}_p^{*T} \phi(x,u) - r_p(x,u,\phi,\overline{Y}_p^*) \end{aligned} \tag{25}$$

Subtracting (25) from (20) yields

$$\begin{aligned} \dot{\tilde{z}}_1 &= -\alpha_1 \tilde{z}_1 + \Delta_1^T \phi(x,u) + r_1(x,u,\phi,\overline{Y}_1^*) \\ &\vdots \\ \dot{\tilde{z}}_p &= -\alpha_p \tilde{z}_p + \Delta_p^T \phi(x,u) + r_p(x,u,\phi,\overline{Y}_p^*) \end{aligned} \tag{26}$$

where $\Delta_i = \overline{Y}_i - \overline{Y}_1^*$ is the parameter estimation error.

In this situation we propose the following control law which is based on DFLS:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} = \begin{bmatrix} \hat{g}_{11}(x) & \cdots & \hat{g}_{1p}(x) \\ \vdots & \ddots & \vdots \\ \hat{g}_{p1}(x) & \cdots & \hat{g}_{pp}(x) \end{bmatrix}^{-1} \left(\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_p \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} - \begin{bmatrix} \overline{Y}_1^T \phi(x,0) \\ \vdots \\ \overline{Y}_p^T \phi(x,0) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} \right) \tag{27}$$

Or equivalently, equation (27) can be written in the compact form:

$$u = \hat{G}(x)^{-1} [\alpha z - \overline{Y}^T \phi(x,0) + v + u_r] \tag{28}$$

where $\hat{G}(x)$ is a static fuzzy logic estimation of $G(x)$, $\phi(x,0) = \phi(x,u)\big|_{u=0}$, and u_r is a robust compensator which defined as

$$u_{ri} = \frac{1}{\lambda_i} B_i^T P_i \underline{e}_i \tag{29}$$

where λ_i, P_i are the solution of the following Riccati-like equation.

$$A_i^T P_i + P_i A_i - Q_i - \left(\frac{2}{\lambda_i} - \frac{1}{\rho^2} \right) P_i B_i B_i^T P_i = 0 \tag{30}$$

Using (27), we can rewrite (12) as

$$\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_p \end{bmatrix} &= \begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix} + \left(G(x) + \hat{G}(x) - \hat{G}(x) \right) G^{-1}(x) \\ &\left(\begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_p \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix} - \begin{bmatrix} \overline{Y}_1^T \phi(x,0) \\ \vdots \\ \overline{Y}_p^T \phi(x,0) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} \right) + \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix} \end{aligned} \tag{31}$$

Using (17), (18), (31), and after straight forward manipulations, it can be easily to obtain

$$\begin{bmatrix} e_1^{(n_1)} + k_{1r_1} e_1^{(n_1-1)} + \cdots + k_{11} e_1 \\ \vdots \\ e_p^{(n_p)} + k_{pr_p} e_p^{(n_p-1)} + \cdots + k_{p1} e_p \end{bmatrix} = \begin{bmatrix} f_1(x) + \alpha_1 z_1 - \bar{Y}_1^T \phi(x,0) \\ \vdots \\ f_p(x) + \alpha_p z_p - \bar{Y}_p^T \phi(x,0) \end{bmatrix} + (G(x) - \hat{G}(x)) \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} + \begin{bmatrix} u_{r1} \\ \vdots \\ u_{r2} \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix} \tag{32}$$

It is clear from (32) that u_{ri} can attenuate the external disturbance and fuzzy approximation errors.

Equation (32) can be written as:

$$e_i^{(n_i)} + k_{ir_i} e_i^{(n_i-1)} + \cdots + k_{i1} e_i = f_i(x) + \alpha_i z_i - \bar{Y}_i^T \phi(x,0) + \sum_{j=1}^{p} \left(g_{ij}(x) - \hat{g}_{ij}(x) \right) u_j - u_{ri} + d_i, \quad i = 1,2,\cdots p \tag{33}$$

The state-space form for (33) can be written as:

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \begin{bmatrix} f_i(x) + \alpha_i z_i - \bar{Y}_i^T \phi(x,0) \\ + \sum_{j=1}^{p} \left(g_{ij}(x) - \hat{g}_{ij}(x) \right) u_j \end{bmatrix} + B_i d_i \tag{34}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{in_i} & -k_{i(n_i-1)} & -k_{i(n_i-2)} & \cdots & -k_{i1} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Using (22) with $u = 0$, we can write

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \left[-\Delta_i^T \phi(x,0) + \sum_{j=1}^{p} (g_{ij}(x) - \hat{g}_{ij}(x)) u_j - r_i(x,u,0,\bar{Y}_i^*) \right] \tag{35}$$

The control gain functions $g_{ij}(x)$ can be approximated by a static fuzzy logic system (6). It follows that

$$g_{ij}(x) = \hat{g}_{ij}(x) + \kappa_{ij} = \bar{Y}_{ij}^T \phi(x) + \kappa_{ij} \tag{36}$$

where $\bar{Y}_{ij}$ is an adjustable parameter vector, $\phi(x)$ is a fuzzy basis function and κ_{ij} is the fuzzy approximation error. Define an optimal parameter estimates $\bar{Y}_{ij}^*$ such that it minimizes the approximation error. Therefore, we can write

$$g_{ij}(x) = \bar{Y}_{ij}^{*T} \phi(x) + \kappa_{ij}^* \tag{37}$$

where κ_{ij}^* is the minimum approximation error. Using (36) and (37), equation (35) can be written as

$$\underline{\dot{e}_i} = A_i e_i + B_i u_{ri} + B_i \left[-\Delta_i^T \phi(x,0) - \sum_{j=1}^{p} \Delta_{ij}^T \phi(x) u_j \right] + B_i w_i \tag{38}$$

where $\Delta_{ij}^T = \bar{Y}_{ij}^T - \bar{Y}_{ij}^{*T}$ and $w_i = \kappa_{ij}^* + r_i(x,u,0,\bar{Y}_i^*)$.

The adaptive laws are chosen as

$$\dot{\bar{Y}}_i = -\eta_i h_i \tilde{z}_i \phi(x,u) + \eta_i \phi(x,0) B_i^T P_i \underline{e}_i \tag{39}$$

and

$$\dot{\bar{Y}}_{ij} = -\eta_{ij} \underline{e}_i^T P_i B_i \phi(x) u_j \tag{40}$$

Theorem 1. Consider an unknown MIMO nonlinear dynamic system (1) which is controlled by (27) and to be identified by the DFLS (20) by adjusting the parameter vectors $\bar{Y}_i^T$ and $\bar{Y}_{ij}^T$ with the adaptive laws (39) and (40) respectively, then the closed loop system possess the following properties

(i) All signals in the closed-loop system are uniformly bounded.

(ii) For a given disturbance attenuation level, the proposed tracking performance index (3) is achieved.

Proof. Choose a Lyapunov function as

$$V = V_1 + \cdots + V_p, \tag{41}$$

$$V_i = \frac{1}{2}\underline{e}_i^T P_i \underline{e}_i + \frac{1}{2}h_i \tilde{z}_i^2 + \frac{1}{2\eta_i}\Delta_i^T \Delta_i + \sum_{j=1}^{p}\frac{1}{2\eta_{ij}}\Delta_{ij}^T \Delta_{ij} \tag{42}$$

Differentiating V, and V_i

$$\dot{V} = \dot{V}_1 + \cdots + \dot{V}_p , \tag{43}$$

Substituting (26) and (38) in (43) after differentiation, we can obtain equation (44) at the bottom of this page.

Using the adaptive laws (39) and (40), equation (44) can be simplified into

$$\begin{aligned}\dot{V}_i \le{}& \underline{e}_i^T\left(A_i^T P_i + P_i A_i - \frac{2}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i \right)\underline{e}_i \\ &- \alpha_i h_i \tilde{z}_i^2 - h_i \tilde{z}_i\, r_i(x,u,\phi,\overline{Y}_i^*) - \frac{1}{2}\left(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i \right)\end{aligned} \tag{45}$$

Using the following triangular inequality for the third term in (45)

$$h_i \tilde{z}_i\, r_i(x,u,\phi,\overline{Y}_i^*) \le \frac{h_i^2 \tilde{z}_i^2}{2\rho^2} + \frac{\rho^2}{2} r_i^2(x,u,\phi,\overline{Y}_i^*) \tag{46}$$

$$\begin{aligned}\dot{V}_i \le{}& -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2\rho_i^2}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \alpha_i h_i \tilde{z}_i^2 \\ &+ \frac{h_i^2 \tilde{z}_i^2}{2\rho^2} + \frac{\rho^2}{2} r_i^2(x,u,\phi,\overline{Y}_i^*) \\ &+ \frac{1}{2}\left(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i \right)\end{aligned} \tag{47}$$

Note that the third and fourth terms are negative in (47).

$$\begin{aligned}\dot{V}_i \le{}& -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2\rho^2}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i \\ &- \left(\alpha_i h_i - \frac{h_i^2}{2\rho^2} \right)\tilde{z}_i^2 + \frac{\rho^2}{2} r_i^2(x,u,\phi,\overline{Y}_i^*) \\ &+ \frac{1}{2}\left(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i \right)\end{aligned} \tag{48}$$

The third term in (48) can be made negative by choosing $h_i \le 2\alpha_i \rho^2$.

$$\begin{aligned}\dot{V}_i \le{}& -\underline{e}_i^T Q \underline{e}_i - \frac{1}{2}\left(\frac{1}{\rho}\underline{e}_i^T P_i B - \rho w_i \right)^2 \\ &+ \frac{\rho^2}{2}\left(r_i^2(x,u,\phi,\overline{Y}_i^*) + w_i^2 \right)\end{aligned} \tag{49}$$

$$\begin{aligned}\dot{V}_i ={}& \frac{1}{2}\dot{\underline{e}}_i^T P_i \underline{e}_i + \frac{1}{2}\underline{e}_i^T P_i \dot{\underline{e}}_i + h_i \tilde{z}_i \dot{\tilde{z}}_i + \frac{1}{\eta_i}\dot{\Delta}_i^T \Delta_i + \sum_{j=1}^{p}\frac{1}{\eta_{ij}}\dot{\Delta}_{ij}^T \Delta_{ij} \\ ={}& \frac{1}{2}\Bigg[\underline{e}_i^T A_i^T P_i \underline{e}_i - \frac{1}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \phi(x,0)^T \Delta_i B_i^T P_i \underline{e}_i - \sum_{j=1}^{p}\phi(x)\Delta_{ij} B_i^T \underline{e}_i u_j + w_i B_i^T P_i \underline{e}_i \\ &+ \underline{e}_i^T P_i A_i \underline{e}_i - \frac{1}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i - \underline{e}_i^T P_i B_i \Delta_i^T \phi(x,0) - \sum_{j=1}^{p}\underline{e}_i^T P_i B_i \Delta_{ij}^T \phi(x) u_j + \underline{e}_i^T P_i B_i w_i \Bigg] \\ &+ h_i \tilde{z}_i \left[-\alpha_i \tilde{z}_i + \Delta_i^T \phi(x,u) + r_i(x,u,\phi,\overline{Y}_i^*) \right] + \frac{1}{\eta_i}\dot{\tilde{Y}}_i^T \Delta_i + \sum_{j=1}^{p}\frac{1}{\eta_{ij}}\dot{\tilde{Y}}_{ij}^T \Delta_{ij} \\ ={}& \underline{e}_i^T\left(A_i^T P_i + P_i A_i - \frac{2}{\lambda_i}\underline{e}_i^T P_i B_i B_i^T \underline{e}_i \right)\underline{e}_i - \alpha_i h_i \tilde{z}_i^2 + \frac{1}{\eta_i}\left(\dot{\tilde{Y}}_i^T + \eta_i h_i \tilde{z}_i \phi^T(x,u) - \eta_i \phi(x,0) B_i^T P_i \underline{e}_i \right)\Delta_i \\ &+ \sum_{j=1}^{p}\frac{1}{\eta_{ij}}\left(\dot{\tilde{Y}}_{ij}^T + \eta_{ij}\underline{e}_i^T P_i B_i \phi^T(x) u_j \right)\Delta_{ij} + h_i \tilde{z}_i\, r_i(x,u,\phi,\overline{Y}_i^*) + \frac{1}{2}\left(w_i B_i^T P_i \underline{e}_i + \underline{e}_i^T P_i B_i w_i \right)\end{aligned} \tag{44}$$

Since

$\frac{1}{2}\left(\frac{1}{\rho}\underline{e}_i^T P_i B - \rho\, w_i\right)^2 \geq 0$, from (49) we obtain

$$\dot{V}_i \leq -\underline{e}_i^T Q \underline{e}_i + \frac{1}{2}\rho^2 \delta_i^2 \tag{50}$$

where $\delta_i^2 = r_i^2(x, u, \phi, \overline{Y}_i^*) + w_i^2$.

After some straightforward manipulations, we can deduce

$$\dot{V}_i \leq -c_i V_i + \mu_i \tag{51}$$

where

$c_i = \min\left\{\lambda, \frac{1}{\eta_i}, \frac{1}{\eta_{ij}}\right\}$ with $\lambda = \frac{\inf \lambda_{\min}(Q_i)}{sub\, \lambda_{\max}(Q_i)}$ and

$\mu_i = \frac{1}{2\rho^2}\sum_{i=1}^{p}\delta_i^2$

From (51) and (43)

$$\dot{V} \leq -cV + \mu \tag{52}$$

where

$c = \sum_{i=1}^{p} c_i$, $\mu = \sum_{i=1}^{p}\mu_i$

This implies that all signals in the closed loop system is bounded. Thus the control objective (i) is realized.

Integrating (50) from $t = 0$ to $t = T$, we have

$$\frac{1}{2}\int_0^T \underline{e}_i^T Q \underline{e}_i dt \leq V_i(0) - V_i(T) + \frac{1}{2}\rho^2 \int_0^T \delta_i^2 dt \tag{53}$$

Since $V_i(T) \geq 0$ we can write (53) as follows:

$$\begin{aligned} &\frac{1}{2}\int_0^T \underline{e}_i^T Q \underline{e}_i dt \leq V_i(0) + \frac{1}{2}\rho^2 \int_0^T \delta_i^2 dt \\ &= \frac{1}{2}\underline{e}_i^T(0) P_i \underline{e}_i(0) + \frac{1}{2}h_i \tilde{z}_i^2(0) \\ &+ \frac{1}{2\eta_i}\Delta_i^T(0)\Delta_i(0) \\ &+ \sum_{j=1}^{p}\frac{1}{2\eta_{ij}}\Delta_{ij}^T(0)\Delta_{ij}(0) + \frac{1}{2}\rho^2\int_0^T \delta_i^2 dt \end{aligned} \tag{54}$$

Let

$Q = diag[Q_1, \cdots, Q_P]$, $P = diag[P_1, \cdots, P_P]$, $\underline{e} = [\underline{e}_1^T, \cdots, \underline{e}_P^T]^T$, $\tilde{z} = [\tilde{z}_1, \cdots \tilde{z}_p]^T$,

$\Delta = \left[\frac{1}{\eta_1}\Delta_1^T, \cdots, \frac{1}{\eta_p}\Delta_p^T, \frac{1}{\eta_{11}}\Delta_{11}^T, \cdots, \frac{1}{\eta_{pp}}\Delta_{pp}^T,\right]^T$, and $\delta = [\delta_1, \cdots, \delta_p]^T$

Then from (54) we obtain

$$\begin{aligned} \frac{1}{2}\int_0^T \underline{e}^T Q \underline{e}\ dt &\leq \frac{1}{2}\underline{e}^T(0) P \underline{e}(0) + \frac{1}{2}h\tilde{z}^T\tilde{z}(0) \\ &+ \frac{1}{2}\Delta^T(0)\Delta(0) + + \frac{1}{2}\rho^2\int_0^T \delta^T\delta\, dt \end{aligned} \tag{55}$$

Thus the control objective (ii) is achieved and the proof of Theorem 1 is completed.

V. Simulation Results

To demonstrate the effectiveness of the proposed scheme, a simulation is performed for the tracking control of a two-link rigid robot manipulator moving in horizontal plane. The dynamics of the robot are described by the following differential equation:

$$\begin{aligned} &\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \\ &\begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \end{aligned} \tag{56}$$

where

$M_{11} = a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2)$

$M_{22} = a_2$

$M_{21} = M_{12} = a_2 + a_3\cos(q_2) + a_4\sin(q_2)$

$h = a_3\sin(q_2) - a_4\cos(q_2)$

with

$a_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$

$a_2 = I_e + m_e l_{ce}^2$

$a_3 = m_e\, l_1\, l_{ce}\cos\delta_e$

$a_4 = m_e\, l_1\, l_{ce}\sin\delta_e$

In the simulation, the following parameter values are used:

$l_1 = 0.1 \quad l_{c1} = 0.5 \quad m_1 = 1.0 \quad I_1 = 0.12$

$I_{ce} = 0.6 \quad \delta_e = 0.6 \quad m_e = 2.0 \quad I_e = 0.25$

Since the inertia matrix M is positive definite, the system can be written as:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (57)$$

Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, $y_1 = x_1$, $y_2 = x_3$, $G(x) = M^{-1}$, $F(x) = M^{-1} \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$

Then, the dynamics of the two-link robotic manipulator can be expressed as:

$$\ddot{y} = F(x) + G(x)u \quad (58)$$

The control objective is to force the system outputs q_1 and q_2 to track the desired trajectories:

$y_{m1} = 0.2\sin(t)$, $y_{m2} = 0.2\sin(t)$

For the DFLS, seven membership functions are selected as follows

$\mu_{F_i^1} = 1/(1 + \exp(-5(x_i + 1.5)))$,

$\mu_{F_i^2} = \exp(-5(x_i + 1)^2)$,

$\mu_{F_i^3} = \exp(-5(x_i + 0.5)^2)$,

$\mu_{F_i^4} = \exp(-5x_i^2)$

$\mu_{F_i^5} = \exp(-5(x_i - 0.5)^2)$,

$\mu_{F_i^6} = \exp(-5(x_i - 1)^2)$,

$\mu_{F_i^7} = 1/(1 - \exp(-5(x_i - 1.5)))$.

For given

$Q_i = diag[10,10]$, $\rho = 0.1,0.2$, $\lambda = 0.01,0.02$.

Solving Riccati equation yields

$$P_i = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix},$$

Choosing

$\alpha_1 = 9, \alpha_2 = 5,\ h_1 = 1000, h_2 = 200,$

$k_{11} = k_{21} = 4, k_{12} = k_{22} = 10,\ \eta_1 = \eta_1 = 1,$

$\eta_{11} = \eta_{22} = 0.1$, $\eta_{12} = 0.11$, $\eta_{21} = 0.21$

Choose initial condition as

$x_1(0) = x_3(0) = 0.1, x_2(0) = x_4(0) = 0$,

and the initial conditions for the adaptive parameters are chosen to be zero. Figs. 3-6 show the simulation results.

VI. CONCLUSIONS

In this paper an adaptive fuzzy control scheme is developed based on DFLS for a class of uncertain nonlinear MIMO systems. DFLS is used to identify the unknown nonlinear system as a whole and an adaptive fuzzy controller is developed. The fuzzy control law is robustfied by an H$^\infty$ compensator to attenuate disturbances and fuzzy approximation errors. The proposed approach guarantees that all signals in the closed loop system are uniformly bounded. Simulation results show the effectiveness of the proposed scheme.

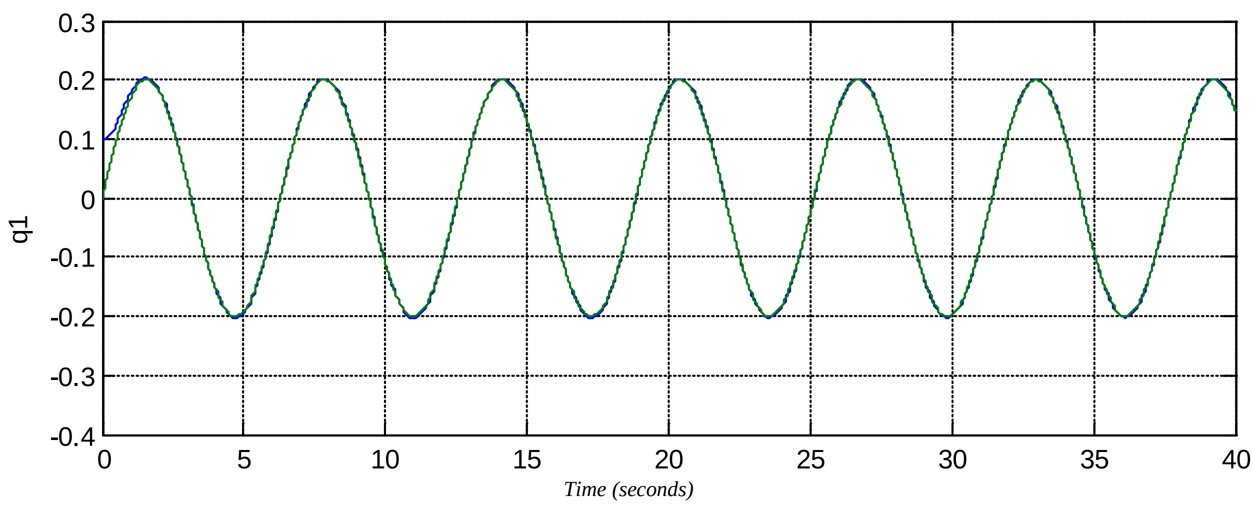

Fig. 3. Tracking curves of q1

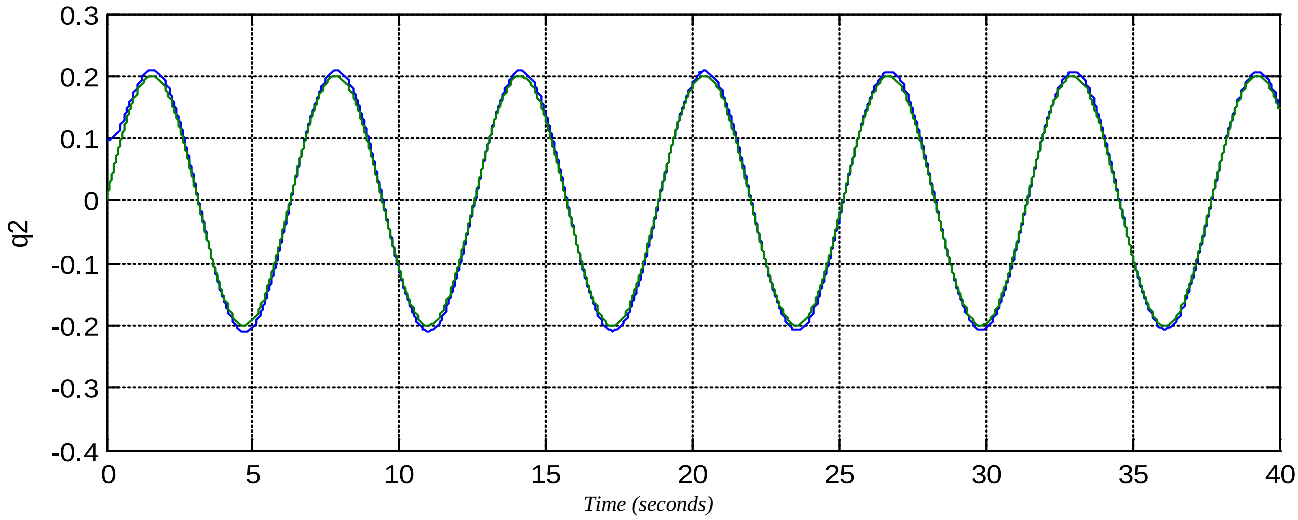

Fig. 4. Tracking curves of q_2

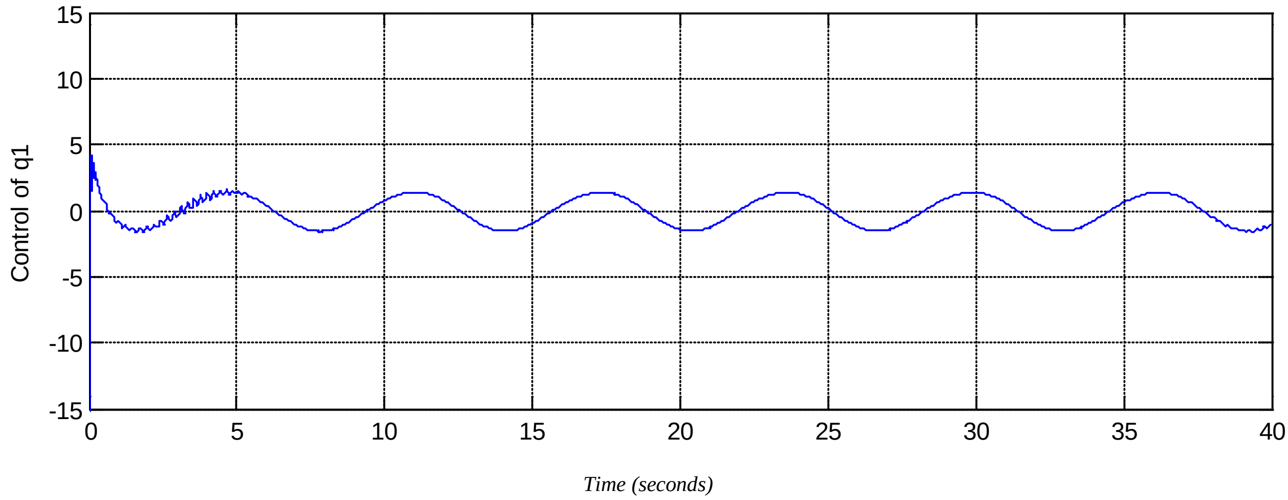

Fig. 5. The control input u_1

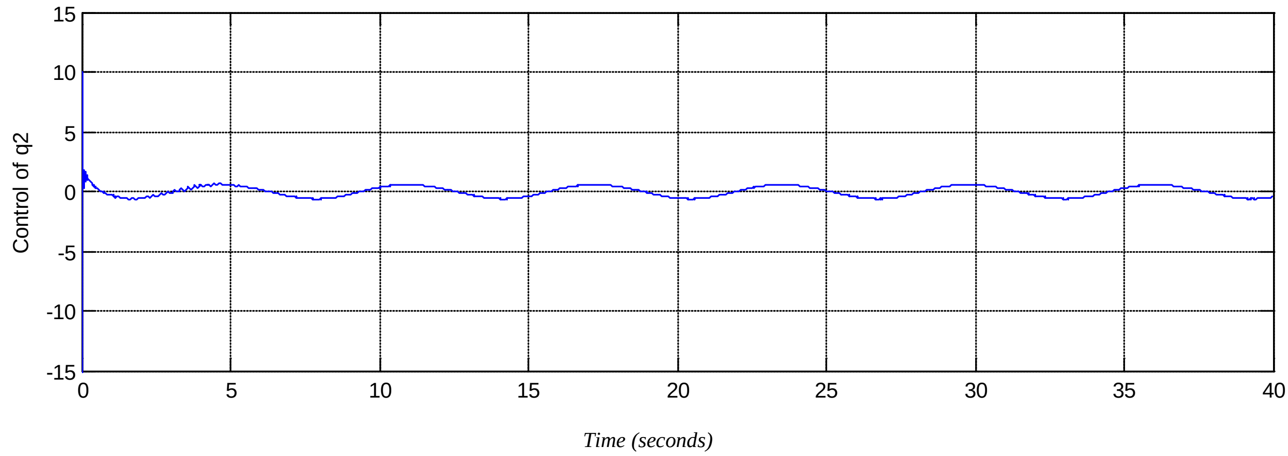

Fig. 6. The control input u_2

ACKNOWLEDGMENT

The author would like to thank Prof. Hasan A. Yousef, of Alexandria University, for his help, as well as the Editor and the reviewers for their inspiring encouragement and constructive comments, which have contributed much to the improvement of the clarity and presentation of this paper.

REFERENCES

[1] Horacio J. Marquez, Nonlinear Control Systems: Analysis and Design, Wiley Interscience, 2003.

[2] J.E. Slotine, W. Li, Applied Nonlinear Control, Prentice-Hall, Englewood Cliffs, NJ, 1991.

[3] I. Kanellakopoulos, P.V. Kokotovic, R. Maritio, An extended direct scheme for robust adaptive nonlinear control, Automatica, vol. 27 pp. 247-255, 1991.

[4] M.M. Polycarpou, P.A. Ioannou, A robust adaptive nonlinear control design, Automatica, vol. 32, pp. 423-427, 1996.

[5] P. Kokotovic, M. Arcak, Constructive nonlinear control: a historical perspective, Automatica, vol. 37, pp. 637-662, 2001.

[6] M. Kristic, I. Kanellakopoulos, P. Kokotovic, Nonlinear and Adaptive Control Design, Wiley Interscience, New York, 1995.

[7] L. X. Wang, Adaptive Fuzzy Systems and Control: Design and Stability Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1994.

[8] B. Chen, C. Lee, Y. Chang, H_∞ tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach, IEEE Transaction on Fuzzy Systems, vol. 4, pp. 32-43, 1996.

[9] A. Poursamad, A. Markazi , Robust adaptive fuzzy control of unknown chaotic systems, Applied Soft Computing, vol. 9, pp. 970–976, 2009.

[10] S. Tong, H. Li, Direct adaptive fuzzy output tracking control of nonlinear systems, Fuzzy Sets and Systems, vol. 128, pp. 107-115, 2002.

[11] S. Tong, J. Tang, T. Wang, Fuzzy adaptive control of multivariable nonlinear systems, Fuzzy Sets and Systems, vol. 111, pp. 153-167, 2000.

[12] S. Tong, H. Li, Fuzzy adaptive sliding-mode control of MIMO nonlinear systems, IEEE Transaction on Fuzzy Systems, vol. 11, pp. 354-360, 2003.

[13] S. Labiod, M. Boucherit, T. Guerra, Adaptive fuzzy control of MIMO nonlinear systems, Fuzzy Sets and Systems, vol. 151, pp. 59-77, 2005.

[14] H. Yousef, M. Hamdy, E. El-Madbouly, and D. Eteim, Adaptive fuzzy semi-decentralized control for a class of large-scale nonlinear systems with unknown interconnections, International Journal of Robust and Nonlinear Control, vol. 16, pp. 687-708, 2006.

[15] H. Yousef, M. Hamdy, E. El-Madbouly, and D. Eteim, Adaptive fuzzy decentralized control for interconnected MIMO nonlinear subsystems, Automatica, vol. 45, pp. 456-462, 2009.

[16] S. Tong, H. Li, and G. Chen, Adaptive Fuzzy Decentralized Control for a Class of Large-Scale Nonlinear Systems, IEEE Transactions on Systems, Man, and Cybernetics - Part B, vol. 34, pp.770-775, 2004.

[17] J. Lee, G. Vukovich, The dynamic fuzzy logic system: nonlinear system identification and application to robotic manipulators, Robotic Systems, vol. 14, pp. 391-405, 1997.

[18] J. Lee, G. Vukovich, Stable identification and adaptive control: a dynamic fuzzy logic system approach, in: W.Pedrycz (Ed.), Fuzzy Evolutionary Computation, Kluwer Academic Publishers, Boston, London, Dordrecht, pp. 223-248, 1997.

[19] O. Murthy, R. Bhatt, N. Ahmed, Extended dynamic fuzzy logic system DFLS based indirect stable adaptive control of nonlinear systems, Applied Soft Computing, vol. 4, pp. 109-119, 2004.